

### INSTRUCTIONS

This is your exam. Complete it either at [exam.cs61a.org](http://exam.cs61a.org) or, if that doesn't work, by emailing course staff with your solutions before the exam deadline.

This exam is intended for the student with email address <EMAILADDRESS>. If this is not your email address, notify course staff immediately, as each exam is different. Do not distribute this exam PDF even after the exam ends, as some students may be taking the exam in a different time zone.

For questions with **circular bubbles**, you should select exactly *one* choice.

- You must choose either this option
- Or this one, but not both!

For questions with **square checkboxes**, you may select *multiple* choices.

- You could select this choice.
- You could select this one too!

**You may start your exam now. Your exam is due at <DEADLINE> Pacific Time.** Go to the next page to begin.

**Preliminaries**

You can complete and submit these questions before the exam starts.

- (a) What is your full name?

- (b) What is your student ID number?

- (c) What is your @berkeley.edu email address?

- (d) Sign (or type) your name to confirm that all work on this exam will be your own. The penalty for academic misconduct on an exam is an F in the course.

**1. (6.0 points) What Would Python Display?**

Assume the following code has been executed.

```
def cut(s):
    this = 1
    for x in s:
        if this:
            this = x
            yield this
        else:
            this = True
```

```
def paste(n):
    yield n
    for x in paste(n + 1):
        yield 2 * x
    yield n
```

```
def copy(t, k):
    return [next(t) for x in range(k)]
```

```
nums = [0, 2, 4, 6, 8]
```

Write the output that would be displayed by printing the result of each expression. If an error occurs, write ERROR.

(a) (2.0 pt) `list(cut(nums))`

- [0, 2, 4, 6, 8]
- [0, 4, 6, 8]
- [1, 0, 2, 4, 6, 8]
- [1, 0, 4, 6, 8]
- [1, 2, 4, 6, 8]
- [2, 4, 6, 8]

(b) (2.0 pt) `list(cut(map(lambda x: x - 4, cut(nums))))`

(c) (2.0 pt) `copy(paste(0), 3)`

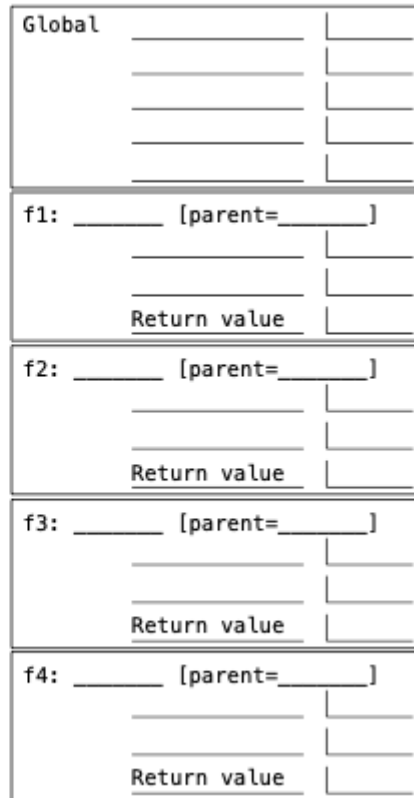
**2. (4.0 points) Line Diagram**

Complete the environment diagram to answer the following questions. Only the questions will be scored.

```

1: b = 6
2: log = []
3:
4: def make_line(a):
5:     b = 1
6:     list(log).append(a)
7:     return lambda x: a * x + b
8:
9: def shift_line(f):
10:    b = 3
11:    log.append([b])
12:    return lambda y: f(y) + b
13:
14: z = shift_line(make_line(2))(0)
15: print(log)
16: print(z)

```



(a) (2.0 pt) What is displayed by `print(log)` on line 15?

- []  
 [3]  
 [[3]]  
 [[[3]]]  
 [2, 3]  
 [2, [3]]  
 [2, [[3]]]  
 [[2], [3]]  
 [[2], [[3]]]

(b) (2.0 pt) What is displayed by `print(z)` on line 16?

- 2  
 4  
 6  
 7  
 9  
 12

**3. (5.0 points) Make Tens**

A list of numbers is easier for a person to sum up if it can be split into groups that all sum to 10. Implement `tens`, which takes a list of positive numbers `s`. It returns `True` if, for every positive integer `i` where  $10 * i \leq \text{sum}(s)$ , there is a positive `k` that satisfies  $\text{sum}(s[0:k]) = 10 * i$ . Otherwise, it returns `False`.

```
def tens(s):
    """Return whether every multiple of 10 less than or equal to sum(s) appears as a prefix of s.
    >>> tens([3, 2, 2, 3, 6, 2, 2, 4, 1, 5, 2]) # sum(s[:4])==10, sum(s[:7])==20, sum(s[:10])==30
    True
    >>> tens([3, 2, 2, 3, 6, 2, 6, 1, 5, 2]) # sum(s[:4])==10, but no slice starting at 0 sums to 20
    False
    """
    t = 0
    for x in s:
        t += x
        if -----:
            (g)
            return -----
            (h)
        if t == -----:
            (i)
            t = -----
            (j)
    return True
```

(a) (1.0 pt) Fill in blank (g).

- `t == 10`  
 `t != 10`  
 `t > 10`  
 `t < 10`

(b) (2.0 pt) Fill in blank (h). Select all correct answers.

- `False`  
 `t % 10 == 0`  
 `t % 10 != 0`  
 `sum(s) % 10 == 0`  
 `sum(s) % 10 != 0`

(c) (1.0 pt) Fill in blank (i).

- `0`  
 `10`  
 `x`

(d) (1.0 pt) Fill in blank (j).

## 4. (13.0 points) Count Misses

## (a) (6.0 points)

Implement the `Counter` class. A `Counter` has a `count` of the number of times `inc` has been invoked on itself or any of its offspring. Its offspring are the `Counters` created by its `spawn` method or the `spawn` method of any of its offspring.

```
class Counter:
    """Counts how many times inc has been invoked on itself or any of its offspring.

    >>> total = Counter()
    >>> odd, even = total.spawn(), total.spawn() # these are offspring of total
    >>> one, three = odd.spawn(), odd.spawn()    # these are offspring of odd and total
    >>> for c in [one, even, three, even, odd, even]:
    ...     c.inc()
    >>> [c.count for c in [one, three, even, odd, total]]
    [1, 1, 3, 3, 6]
    """
    def __init__(self, parent=None):
        self.parent = parent
        -----
        (a)

    def inc(self):
        self.count += 1
        -----:
        (b)
        -----
        (c)

    def spawn(self):
        return -----
        (d)
```

i. (1.0 pt) Fill in blank (a).

ii. (1.0 pt) Fill in blank (b).

- if parent is not None:
- if self.parent is not None:
- while parent is not None:
- while self.parent is not None:
- for p in parent:
- for p in self.parent:

iii. (2.0 pt) Fill in blank (c).

- p += 1
- p.count += 1
- p.inc()
- p.count.inc()
- parent += 1
- parent.count += 1
- parent.inc()
- parent.count.inc()
- self.parent += 1
- self.parent.count += 1
- self.parent.inc()
- self.parent.count.inc()

iv. (2.0 pt) Fill in blank (d).

- self.parent
- self.parent.spawn()
- self.spawn()
- Counter()
- Counter().spawn()
- Counter(self)
- Counter(self.count)

**(b) (7.0 points)**

Implement the `MissDict` class. A `MissDict` has a dictionary `d`. Its `get` method takes an iterable `keys`, returns a list of all values in `d` that correspond to those `keys`, and counts the number of `keys` that did not appear in `d` (called *misses*). Printing a `MissDict` displays a fraction in which:

- The numerator is the number of misses during all calls to `get` for that particular `MissDict` instance.
- The denominator is the number of misses during all calls to `get` for any `MissDict` instance.

Assume `Counter` is implemented correctly.

```
class MissDict:
    """Has a dict, gets a list of values for an iterable of keys,
    and counts the number of keys that are not in the dict.

    >>> double = MissDict({1: 2, 2: 4, 3: 6, 5: 10})
    >>> half = MissDict({2: 1.0, 3: 1.5, 4: 2.0})
    >>> double.get([1, 3, 5, 2, 4]) # No value for key 4 (1 miss)
    [2, 6, 10, 4]
    >>> double.get([5, 4, 3, 0, 4]) # No value for keys 4 or 0 or 4 (3 misses)
    [10, 6]
    >>> half.get([1, 3, 5, 2, 4]) # No value for keys 1 or 5 (2 misses)
    [1.5, 1.0, 2.0]
    >>> print(double) # double had 4 misses & half had 2 misses
    4/6 of the misses
    """
    misses = Counter()
    def __init__(self, d):
        assert isinstance(d, dict)
        self.d = d
        self.misses = -----
                        (e)

    def get(self, keys):
        result = []
        for k in keys:
            if k in self.d:
                -----
                (f)
            else:
                -----
                (g)
        return result

    def __str__(self):
        return f'----- of the misses'
        (h)
```

i. (2.0 pt) Fill in blank (e). Select all correct answers.

- `MissDict.spawn()`
- `MissDict.misses.spawn()`
- `Counter()`
- `Counter(MissDict)`
- `Counter(MissDict.misses)`

ii. (2.0 pt) Fill in blank (f).

iii. (1.0 pt) Fill in blank (g).

- `misses.inc()`
- `misses.count += 1`
- `self.misses.inc()`
- `self.misses.count += 1`
- `MissDict.misses.inc()`
- `MissDict.misses.count += 1`

iv. (2.0 pt) Fill in blank (h).

- `{misses.count / MissDict.misses.count}`
- `{misses.count} / {MissDict.misses.count}`
- `{misses.count / self.MissDict.misses.count}`
- `{misses.count} / {self.MissDict.misses.count}`
- `{self.misses.count / misses.count}`
- `{self.misses.count} / {misses.count}`
- `{self.misses.count / MissDict.misses.count}`
- `{self.misses.count} / {MissDict.misses.count}`
- `{self.misses.count / self.MissDict.misses.count}`
- `{self.misses.count} / {self.MissDict.misses.count}`

**5. (16.0 points) Promotion to CS 61B**

**Definition.** Given a sequence  $s$  and positions (indices)  $i$  and  $j$  in  $s$  with  $i < j$ , *promoting* an element to  $i$  from  $j$  means reordering  $s$  so that the element originally at position  $j$  is now at position  $i$ , all elements originally positioned between  $i$  and  $j$  increase their position (index) by one, and all other elements stay where they are. The beginning of  $s$  is position 0. For example, in the list  $[30, 60, 90, 120, 150, 180]$ , promoting to 2 from 4 would place the number 150 (original position 4) just before 90 (originally position 2) and increases the positions of both 90 and 120 by one, resulting in  $[30, 60, 150, 90, 120, 180]$ .

**(a) (4.0 points)**

Implement `promote`, which takes a list of numbers  $s$  and two non-negative integers  $i$  and  $j$ . It returns a new list promoting to  $i$  from  $j$  in  $s$ . Do not mutate  $s$ .

**Hint:** For any list  $s$ ,  $s[\text{len}(s):]$  evaluates to  $[]$ .

```
def promote(s, i, j):
    """Return a list in which s[j] is at index i without mutating s.

    >>> promote([3, 6, 9, 12, 15, 18], 2, 4)
    [3, 6, 15, 9, 12, 18]
    >>> promote([3, 6, 9, 12, 15, 18], 0, 4)
    [15, 3, 6, 9, 12, 18]
    """
    assert i >= 0 and i < j and j < len(s)
    return s[:i] + _____ + _____ + s[_____:]
                    (a)         (b)         (c)
```

**i. (2.0 pt)** Fill in blank (a). **Select all correct answers.**

- `s[j]`
- `[s[j]]`
- `list(s[j])`
- `s[j:j]`
- `s[j:j+1]`

**ii. (1.0 pt)** Fill in blank (b).

- `s[i:j]`
- `s[i+1:j]`
- `s[i:j+1]`
- `s[i+1:j+1]`

**iii. (1.0 pt)** Fill in blank (c).

- `i`
- `i+1`
- `j`
- `j+1`

**(b) (4.0 points)**

Implement `promotions`, which takes two lists of numbers `s` and `t` that have the same elements, possibly in different orders. It returns the minimum number of promotions that must be applied to `s` so that it has the same order as `t`. Assume `promote` is implemented correctly.

```
def promotions(s, t):
    """Return the minimum times promote must be called to start from s and return t.

    >>> promotions([2, 4, 6, 8, 10, 12],
    ...           [2, 6, 8, 4, 12, 10]) # promote (1, 2) then (2, 3) then (4, 5)
    3
    >>> promotions([6, 1, 6, 1, 6, 1],
    ...           [1, 1, 6, 6, 1, 6])   # promote (0, 1) then (1, 5)
    2
    >>> promotions([1, 2, 3], [1, 2, 3])      # no promotions needed
    0
    >>> promotions([1, 2, 1, 2], [2, 1, 2, 1]) # promote (0, 3)
    1
    """
    assert sorted(s) == sorted(t) # Check that s & t have the same elements (disregarding order)
    if len(s) == 0:
        return 0
    elif _____:
        (d)
        return promotions(s[1:], t[1:])
    else:
        return 1 + min([promotions(_____, t[1:]) for j in range(1, len(s)) if _____])
                        (e)                               (f)
```

i. (1.0 pt) Fill in blank (d).

- `s == t`
- `s != t`
- `s[0] == t[0]`
- `s[0] != t[0]`
- `sorted(s) == t`
- `s == sorted(t)`

ii. (2.0 pt) Fill in blank (e).

iii. (1.0 pt) Fill in blank (f).

- `s[0] == t[j]`
- `s[1] == t[j]`
- `s[j] == t[0]`
- `s[j] == t[1]`

**(c) (8.0 points)**

Implement `promote_link`, which takes a `Link` instance `s` (a non-empty linked list) and two non-negative integers `i` and `j` with `i < j` and `j` less than the length of `s`. It mutates `s` by promoting to `i` from `j` and then returns `s`.

The `Link` class appears on Page 2 (left side) of the Midterm 2 Study Guide.

```
def promote_link(s, i, j):
    """Mutate linked list s so that the item at index j is at index i.

    >>> a = Link(3, Link(6, Link(9, Link(12, Link(15, Link(18))))))
    >>> print(promote_link(a, 2, 4))
    <3 6 15 9 12 18>
    >>> print(promote_link(a, 0, 4))
    <12 3 6 15 9 18>
    >>> promote_link(a, 1, 3) is a
    True
    """
    assert i >= 0 and i < j
    if i > 0:
        -----
        (g)
    else:

        insert, tail = s.first, s.rest

        while j > 0:

            ----- # Hint: use multiple assignment: ___ , ___ = ___ , ___
            (h)

            tail, j = tail.rest, j-1

            ----- = insert
            (i)

        return -----
        (j)
```

**i. (2.0 pt)** Fill in blank (g).

- `promote_link(s.rest, i-1, j)`
- `return promote_link(s.rest, i-1, j)`
- `promote_link(s.rest, i, j-1)`
- `return promote_link(s.rest, i, j-1)`
- `promote_link(s.rest, i-1, j-1)`
- `return promote_link(s.rest, i-1, j-1)`

**ii. (2.0 pt)** Fill in blank (h). **Hint:** Use multiple assignment: `___ , ___ = ___ , ___`

iii. (1.0 pt) Fill in blank (i).

- s.first
- s.rest.first
- tail.first
- tail.rest.first

iv. (1.0 pt) Fill in blank (j).

- s
- tail
- Link(insert, tail.rest)
- Link(s.first, tail.rest)

v. (2.0 pt) What is displayed by the call to print in this code?

```
odds = Link(3, Link(5, Link(7, Link(9))))
for i in range(3):
    promote_link(odds, 0, 3)
```

```
print(odds)
```

- <3 5 7 9>
- <3 9 7 5>
- <5 7 9 3>
- <5 9 7 3>
- <9 7 5 3>
- <9 3 5 7>

**6. (12.0 points) Fresh Produce****(a) (5.0 points)**

Implement `products`, which takes a `Tree` instance `t` with positive integer labels and a positive integer `n`. It returns `True` if every path from the root of `t` to a leaf has labels that equal `n` when multiplied together.

The `Tree` class appears on Page 2 (left side) of the Midterm 2 Study Guide.

```
def products(t, n):
    """Return whether the product of labels along every root-to-leaf path is n.

    >>> products(Tree(1, [Tree(2, [Tree(3)])], Tree(6))), 6)
    True
    >>> products(Tree(1, [Tree(2, [Tree(3)])], Tree(6))), 12)
    False
    >>> products(Tree(1, [Tree(2, [Tree(3)])], Tree(5))), 6)
    False
    >>> products(Tree(1, [Tree(5, [Tree(2)])], Tree(12))), 12)
    False
    """
    assert type(n) == int
    if t.is_leaf():
        return _____
            (a)
    if _____:
        (b)

        return False

    return _____
        (c)
```

**i. (1.0 pt)** Fill in blank (a).

- True
- False
- `n == t.label`
- `n % t.label == 0`
- `n % t.label > 0`

**ii. (1.0 pt)** Fill in blank (b).

- `n != t.label`
- `n < t.label`
- `n > t.label`
- `n % t.label > 0`

**iii. (3.0 pt)** Fill in blank (c). **Important:** The second argument to a call to `products` must be an integer (`int`).

## (b) (7.0 points)

**Definition.** An *increasing sequence* is a sequence of integers in which each element after the first is larger than the previous element.

Implement `produce`, which takes a positive integer `n`. It returns a `Tree` of positive integers in which:

- The product of the labels along every root-to-leaf path is `n`,
- Every increasing sequence of integers starting with 1 that has product `n` is a root-to-leaf path, and
- Every sequence of siblings (nodes with a common parent) is an increasing sequence.

```
def produce(n):
    """Return the largest tree in which the labels for every root-to-label path
    are increasing and have product n. Put all siblings in increasing order.
```

```
>>> produce(12)
Tree(1, [Tree(2, [Tree(6)]), Tree(3, [Tree(4)]), Tree(12)])
>>> print(produce(24)) # Paths are 1-2-3-4, 1-2-12, 1-3-8, 1-4-6, and 1-24
```

```
1
  2
    3
      4
        12
      3
        8
      4
        6
      24
```

```
"""
```

```
def grow(t, x):
    for k in range(_____, x + 1):
        (d)

        if _____ % k == 0:
            (e)

            branch = _____
                (f)

            if _____:
                (g)

                t.branches.append(branch)

    return t

return grow(Tree(1), n)
```

i. (1.0 pt) Fill in blank (d).

- 0
- 1
- x
- t.label
- t.label + 1

ii. (1.0 pt) Fill in blank (e).

- x
- n
- t.label
- (t.label // x)
- (t.label // k)

iii. (2.0 pt) Fill in blank (f).

- Tree(k)
- Tree(x)
- grow(Tree(k), x)
- grow(Tree(x), k)
- grow(Tree(k), x // k)
- grow(Tree(x), x // k)

iv. (3.0 pt) Fill in blank (g).

**7. (14.0 points) A Pair of Schemes****(a) (4.0 points)**

Implement the Scheme procedure `all-pairs`, which takes a procedure `f` and a list `s`. It returns `#t` if `(f x y)` is `#t` for every pair of adjacent elements `x` and `y` in `s`. Assume `f` always returns either `#t` or `#f`.

```
(define (inc x y) (= (+ x 1) y)) ; Whether x+1 equals y

;;; Return #t if (f x y) is #t for every pair of adjacent values (x, y) in list s.
;;;
;;; scm> (all-pairs inc '(3 4 5 6 7 8))
;;; #t
;;; scm> (all-pairs inc '(3 4 5 8 7 8))
;;; #f
;;; scm> (all-pairs inc '(3))
;;; #t
(define (all-pairs f s)
  (or (null? s) (null? (cdr s))
      (and _____ (all-pairs f _____ ))))
           (a)                (b)
```

**i. (3.0 pt)** Fill in blank (a). **Select all correct answers.**

- `(car s) (cdr s)`
- `(car s) (car (cdr s))`
- `f (car s) (cdr s)`
- `f (car s) (car (cdr s))`
- `f((car s) (cdr s))`
- `f((car s) (car (cdr s)))`
- `(f (car s) (cdr s))`
- `(f (car s) (car (cdr s)))`
- `(apply f (car s) (cdr s))`
- `(apply f (car s) (car (cdr s)))`

**ii. (1.0 pt)** Fill in blank (b).

- `s`
- `(cdr s)`
- `(cdr (cdr s))`
- `(cons (car s) (cdr s))`
- `(cons (car s) (cdr (cdr s)))`

**(b) (6.0 points)**

Implement the Scheme procedure `show-pairs`, which takes a list `s` and returns a list of every pair of adjacent elements in `s`. A pair is a two-element list.

```
;;; Return a list of every pair of adjacent elements in list s.
;;;
;;; scm> (show-pairs '(3 5 7 9 11 13))
;;; ((3 5) (5 7) (7 9) (9 11) (11 13))
(define (show-pairs s)
  (if (or (null? s) (null? (cdr s))) nil
      ( ( _____ _____ (show-pairs _____ ) )))
        (a)      (b)                (c))
```

i. (1.0 pt) Fill in blank (a).

- car
- cdr
- cons
- list
- append

ii. (3.0 pt) Fill in blank (b).

iii. (1.0 pt) Fill in blank (c).

- s
- (cdr s)
- (cdr (cdr s))
- (cons (car s) (cdr s))
- (cons (car s) (cdr (cdr s)))

iv. (1.0 pt) What order of growth describes the time it takes to execute `(unpair s)` in terms of the length of the input list `s`, assuming that `car`, `cdr`, `cons`, and `null?` are all constant-time operations?

```
(define (unpair s) (cond ((null? s) nil)
                        ((null? (cdr s)) (car s))
                        (else (cons (car (car s)) (unpair (cdr s))))))
```

- constant
- logarithmic
- linear
- quadratic
- exponential

**(c) (4.0 points)**

Implement the Scheme procedure `all-pairs-exp`, which takes a procedure name `proc-name` (a symbol) and a list `s`. It returns an `and` expression that calls the procedure named by `proc-name` on every adjacent pair of elements in `s`. Assume `show-pairs` is implemented correctly.

```
;;; Return an and expression that calls the procedure called proc-name on
;;; every adjacent pair of elements in s.
;;;
;;; scm> (all-pairs-exp 'inc '(3 4 5 6 7 8))
;;; (and (inc 3 4) (inc 4 5) (inc 5 6) (inc 6 7) (inc 7 8))
;;; scm> (eval (all-pairs-exp 'inc '(3 4 5 6 7 8 )))
;;; #t
(define (all-pairs-exp proc-name s)
  ( _____ (map _____ (show-pairs s))))
  (a)          (b)
```

i. (1.0 pt) Fill in blank (a).

- `and`
- `'and`
- `cons and`
- `cons 'and`

ii. (3.0 pt) Fill in blank (b).



- (c) **(2.0 pt)** Fill in blank (c). You may write **AND** to continue the **WHERE** clause (but you don't have to). You may also include other clauses such as **GROUP BY**, **ORDER BY**, **HAVING**, and **LIMIT** (but you don't have to).

**9. (0.0 points) Ape Pull Us**

These two A+ questions are not worth any points. They can only affect your course grade if you have a high A and might receive an A+. Finish the rest of the exam first!

- (a) Implement `all-next`, a macro that takes an expression `x-expr` and a list of numbers. It returns `#t` if every element in `s` besides the first is equal to the value of `x-expr` when `x` is bound to the previous element in `s`. It returns `#f` otherwise. Assume `x-expr` does not contain the symbol `y`.

You may call `all-pairs` from Q6. **Important:** The template has a quasiquote before the blank.

```
;;; Return whether every value in s that follows another value x is equal
;;; to the x-expr evaluated when x is the previous value.
;;;
;;; scm> (all-next (+ x 2) '(3 5 7 9 11 13))
;;; #t
;;; scm> (all-next (* x 2) '(2 4 8 16 32 64))
;;; #t
;;; scm> (all-next (+ x 2) '(3 5 7 8 11 13))
;;; #f
;;; scm> (all-next (+ x 4) '(3 5 7 9 11 13))
;;; #f
(define-macro (all-next x-expr s) ` _____ )
```

- (b) Implement `make_tens` by filling in the blank in `repromote`. The `make_tens` function takes a list of numbers `s`. It returns a list `t` of the same elements reordered so that `tens(t)` returns `True`. Return an order that requires the fewest promotions to reorder `s` into `t`. You may use `tens` from Q2 and `promote` from Q4.

```
def make_tens(s):
    """Return a list t for which tens(t) is true and promotions(s, t) is as small as possible.
    If there is no reordering t of s for which tens(t) is true, return None.
    >>> make_tens([4, 2, 2, 2, 4, 6, 1, 3, 3, 2, 5]) # promote (6, 7), (7, 9), and (8, 10)
    [4, 2, 2, 2, 4, 6, 3, 2, 5, 1, 3]
    >>> make_tens([4, 2, 2, 2, 4, 5, 1, 4, 3, 4]) # promote (2, 4), (4, 7), (5, 9), and (8, 9)
    [4, 2, 4, 2, 4, 4, 2, 5, 3, 1]
    """
    for t in promote(s):
        if tens(t):
            return t
def promote(s):
    for k in range(len(s)):
        yield from repromote(s, k)
def repromote(s, k):
    if k == 0:
        yield s
    elif len(s) > 1:
        for j in range(len(s)):
            for rest in _____:
                yield [s[j]] + rest
```

**No more questions.**